Nonrelativistic electromagnetic surface waves: Dispersion properties in a magnetized dusty electron-positron plasma

Sang-Hoon Cho,[1,*] Hee J. Lee,[2] and Yong-soo Kim[1]
[1]Department of Nuclear Engineering, Hanyang University, Seoul 133-791, Korea
[2]Department of Physics, Hanyang University, Seoul 133-791, Korea
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Nonrelativistic electromagnetic surface waves propagating on the plane interface between dusty electron-positron plasma and vacuum are investigated by specular reflection procedure. In the presence of an applied magnetic field ($\mathbf{B}_0 = B_0\hat{y}$) directed perpendicular to both the interface normal and the wave vector, transverse electromagnetic modes are studied in terms of the dispersion relation. The analytic modes are derived and discussed with the aid of some numerical analysis. The cold electromagnetic surface wave dispersion relation considering the effect of dust particle shows that possible modes appear only when the normalized frequency ($\overline{\omega}$) and the wave vector ($\overline{K}$) satisfy the condition $\overline{\Omega} < \overline{\omega} < \overline{\Omega} + (1+\delta)/2$ and $\overline{K} > \overline{\Omega}$, where $\delta (= n_{0-}/n_{0+})$ is the parameter of charge imbalance in the plasma and $\overline{\Omega}$ is the normalized cyclotron frequency.

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I. INTRODUCTION

It is widely thought and observed that electron-positron (hereafter, the terminology electron-positron is referred to as e^+e^-) plasmas, which are composed of fully ionized particles with same mass and opposite charge, have appeared in the early Universe [1,2], and are frequently encountered in active galactic nuclei [3] and in pulsar magnetospheres [4,5], Van Allen radiation belts, and near the polar cap of fast rotating neutron stars [6–9] as well as in semiconductor plasmas [10]. However, up until now, the research of theoretical and experimental e^+e^- plasma system is less developed and restricted to relativistic case. Surprisingly, recent experiments have presented the possibility of creating a nonrelativistic e^+e^- plasma in the laboratory [11]. This possibility of creating nonrelativistic e^+e^- plasma source in the laboratory is open up to not only the understanding of physical phenomena, but also the development of another plasma source in the microwave discharge using surface waves, so-called surface wave plasma [12]. For this reason, recently, there has been a great deal of interest in studying linear as well as nonlinear wave motion in such plasmas. Stewart and Laing [13] researched linear wave propagation in the region of bulk e^+e^- plasma, and Tsytovich and Wharton [14] investigated preliminary theoretical results and presented an idea for a magnetic mirror device for e^+e^- system experiments. Also, Iwamoto and Avinash *et al.* [15] used a kinetic approach for analysis of bulk waves in an e^+e^- plasma, and Popel, Vladimirov, and Shukla [16] have investigated soliton problem in electron-positron-ion plasmas. However, most of astrophysical plasmas as well as plasma of processing application usually contains negatively charged dust particles with electron and ion, and also such plasma are bounded by vacuum (or another materials). The dusty plasmas, which is characterized by large-size (in the range of 10 μm to 100 μm) particles and usually highly charged ($Q_d \sim 10^3 - 10^4$) due to a variety of processes including plasma currents, photoelectric effects, secondary emissions, etc., are encountered in space, ranging from earth ionosphere to interstellar clouds, and in the laboratory situations, ranging from plasma processing to fusion devices. Obviously, the plasma model including dust particle effect is more realistic than the other conventional pure electron-ion plasma model or pure e^+e^- plasma model. It has been found that charged dust grains can modify the wave dispersion, instabilities, and wave scattering, etc. [17–21].

The main purpose of this paper is to understand the electromagnetic surface wave dispersion properties of a dusty e^+e^- plasmas in the presence of an applied magnetic field ($\mathbf{B}_0 = B_0\hat{y}$). The propagation of surface waves on the interface between a vacuum and a plasma has attracted much attention because of various technological applications as well as its relation to laser fusion and astrophysical problems. Despite this, theoretical research of surface wave phenomena, which include the effect of dust grains has been less developed for the e^+e^- plasma system. The reason why research about theoretical surface wave problems are less investigated is that it is not easy to solve and analyze because of the difficulty of finding correct boundary conditions. In order to derive the surface wave dispersion relation, we use specular reflection procedure [22–25] which solves the wave equation by Fourier transform, treating the semi-infinite problem by appropriately extending the field components into an undefined region. The choice of correct boundary condition for solving the problem by the plasma and vacuum system demands extreme caution. However, the advantage of this method is that it needs only one boundary condition (the continuity of tangential electric field) and automatically takes care of the presence of linear surface charge or current.

In this paper, we assume that the interface between plasma and vacuum is sharp: the physical transition layer from plasma to vacuum is much shorter than any characteristic length involved in the problem. The assumption, which is called sharp boundary model, may be less well justified due to the interaction between the antiparticle and the wall

*Electronic address: shcho@newton.hanyang.ac.kr

material, however, we neglect such interactions in this problem. In this paper, we discussed the surface wave of the transverse magnetic mode propagating on dusty e^+e^- plasmas bounded by vacuum, in the presence of an applied magnetic field. Also, the analytic dispersion modes which is modified by dust particle effect are derived using charge imbalance parameter δ and discussed for some limiting cases with the help of numerical work. This paper is organized as follows. Section II deals with the multifluid basic equations. In Sec. III, electromagnetic surface wave modes in magnetized dusty e^+e^- plasmas are discussed in terms of the dispersion relation. This research is summarized in Sec. IV.

II. MODEL AND BASIC EQUATIONS

The presence of dust particles significantly affect both plasma parameters and collective processes in this system. In particular, charged dust grains can effectively collect electrons and positrons from the background plasma. These equilibrium system consists of charged dust particles, positrons, and electrons, which satisfy overall charge neutrality condition

$$n_+ = n_- + Z_d n_d, \tag{1}$$

where the subscript $+$, $-$, and d are the species index ($+$ for the positron, $-$ for the electron, and d for the dust particles, respectively). The fluid equations appropriate to an e^+e^- plasma consist of the usual continuity and momentum equations for each species, supplemented by Maxwell's equations. We assume that the plasma occupies the region $x>0$, with the plane interface $x=0$ bounded by vacuum ($x<0$) and the waves are propagated in the z direction, so all the wave quantities are represented in a form $\sim e^{ikz-i\omega t}$. We begin with the following basic equations.

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} = \frac{q_\alpha}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{B}\right), \tag{2}$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \tag{3}$$

$$\nabla \cdot \mathbf{E} = 4\pi e(n_+ - n_-), \tag{4}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{5}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{6}$$

where the subscript α is the species index, m the electron mass, $\mathbf{v}$ the fluid velocity, c the speed of light, and $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic fields, respectively.

III. ELECTROMAGNETIC SURFACE WAVE MODES IN MAGNETIZED PLASMAS

In this section, we show that the dispersion relation of electromagnetic wave in a cold plasma can be derived by specular reflection procedure. It is well known that electromagnetic field equations can be split into two independent subsystems, TM and TE modes, of which only the TM modes admit surface wave solutions. In the present geometry TM mode surface wave consists of the field components (E_x, E_z, B_y) and the rest of the components will be put equal to zero [26]. In order to obtain the specular reflection solution of the TM mode surface wave varying as $\sim e^{ik_z z - i\omega t}$, we shall apply the following conditions:

$$\begin{aligned} E_x(x) &= -E_x(-x) \\ E_z(x) &= E_z(-x) \\ B_y(x) &= -B_y(-x) \end{aligned} \tag{7}$$

The conditions of Eq. (7) are not boundary conditions: they are mathematical recipe which are needed to extend artificially the domain of plasma into the region $x<0$ so that Vlasov and Maxwell equations are invariant under the reflection $x \to -x$ and $v_x \to -v_x$ specular reflection condition). But the physical boundary conditions, i.e., the continuity of tangential components of the fields $\mathbf{E}$ and $\mathbf{B}$ for $x=0$, should be enforced to match the vacuum and plasma solutions. Using Eq. (7), we get the Fourier-transformed Maxwell equation which is extended over to the entire space $-\infty < x < \infty$

$$k_x E_z(k_x) - k_z E_x(k_x) + \frac{w}{c} B_y(k_x) = 0, \tag{8a}$$

$$i k_z B_y(k_x) - i\frac{w}{c} E_x(k_x) + \frac{4\pi}{c} J_x(k_x) = 0, \tag{8b}$$

$$i k_x B_y(k_x) - \frac{a}{\pi} + i\frac{w}{c} E_z(k_x) - \frac{4\pi}{c} J_z(k_x) = 0, \tag{8c}$$

where $a \equiv B_y(x=0^+)$. The current J_x and J_z are determined in terms of the extended electric and magnetic fields as if the entire space ($-\infty < x < \infty$) were filled out by the same plasma. We shall assume that external magnetic field $\mathbf{B}_0 = B_0\hat{y}$ (directed perpendicular both to the interface normal and the wave vector) and introduce the cyclotron frequency $\Omega \equiv eB_a/mc$. Then linearized e^+ and e^- equation of motion and continuity equation give

$$-iw\mathbf{v}_\alpha = \frac{e_\alpha}{m}\mathbf{E} + \Omega_\alpha \mathbf{v}_\alpha \times \hat{y}, \tag{9}$$

$$n_\alpha = \frac{n_{\alpha 0}}{w}\mathbf{k}\cdot\mathbf{v}_\alpha, \tag{10}$$

where $\alpha = e,p$.

Using the above equations, the velocity components v_x and v_z can be written in terms of the field components E_x, E_z, and B_y, and thus we have the following expressions for the components of the current $\mathbf{J} = eN(\mathbf{v}_p - \mathbf{v}_e)$

$$J_x = \frac{iw}{4\pi(w^2 - \Omega^2)}(w_{p+}^2 + w_{p-}^2)E_x, \tag{11}$$

$$J_z = \frac{iw}{4\pi(w^2 - \Omega^2)}(w_{p+}^2 + w_{p-}^2)E_z. \tag{12}$$

By substituting Eqs. (11) and (12), Eqs. (8a)–(8c) can be arranged into the form,

$$\begin{pmatrix} k_x & -k_z & \frac{w}{c} \\ \xi k_x k_z & -\varepsilon_L+\xi\Omega^2 & \frac{ck_z}{w} \\ \varepsilon_L-\xi\Omega^2 & -\xi k_x k_z & \frac{ck_x}{w} \end{pmatrix} \begin{pmatrix} E_z \\ E_x \\ B_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{iac}{\pi\omega} \end{pmatrix}, \tag{13}$$

where $\varepsilon_L \equiv 1-\omega_{p+}^2+\omega_{p-}^2/\omega^2$, $\xi\equiv\omega_{p+}^2+\omega_{p-}^2/\omega^2(\omega^2-\Omega^2)$, and $\omega_{p\pm}^2\equiv 4\pi e^2 n_{p\pm}/m$ have been introduced.

After straightforward algebra, we arrive at the following solution of Eq. (13)

$$E_z(k_x)=\frac{ia}{\pi D}\left\{\frac{c^2}{\omega^2}k_z^2-\varepsilon_L+\xi\Omega^2\right\}, \tag{14}$$

$$E_x(k_x)=\frac{iac^2}{\pi w^2 D}k_x k_z, \tag{15}$$

$$B_y(k_x)=\frac{iack_x}{\pi w D}\{\varepsilon_L-\Omega^2\xi\} \tag{16}$$

where

$$D\equiv\frac{c}{\omega}\{\varepsilon_L-\omega^2\xi\}\left\{\frac{\omega^2}{c^2}(\varepsilon_L-\omega^2\xi)-k^2\right\}$$

is the determinant of the matrix in Eq. (13).

The vacuum solutions are obtained from Eqs. (14) to (16) by replacing $a\to -a$ and putting $\xi=\omega_p=0$. Then, the resulting equations are easily inverted to yield

$$E_z(x)=\frac{iac}{\omega}\lambda e^{\lambda x}, \tag{17}$$

$$E_x(x)=\frac{ac}{\omega}k_z e^{\lambda x}, \tag{18}$$

$$B_y(x)=ae^{\lambda x}, \tag{19}$$

where $\lambda\equiv\sqrt{k_z^2-\omega^2/c^2}$ and $x<0$.

The boundary values of the field components, i.e., the values at $x=0^+$, which are our primary concern, are obtained by operating on Eqs. (14) to (16) with $\int_{-\infty}^{\infty}dk_x$ and by evaluating the integral by the residue theorem. Then, picking up the residues at simple pole $k_x=i\eta$, we get

$$E_z(x=0^+)=-\frac{ia\omega}{c}\frac{\eta}{k_z^2-\eta^2}, \tag{20}$$

$$E_x(x=0^+)=\frac{a\omega}{c}\frac{k_z}{k_z^2-\eta^2}, \tag{21}$$

$$B_y(x=0^+)=a, \tag{22}$$

where

$$\eta\equiv\left\{k_z^2-\frac{\omega^2}{c^2}\left(1-\frac{\omega_{p+}^2+\omega_{p-}^2}{\omega^2-\omega^2}\right)\right\}^{1/2}.$$

These values should be compared with the vacuum side limits of Eqs. (17)–(19). Using Eqs. (17) and (20) for the condition $E_z(x=0^+)=E_z(x=0^-)$ yields the linear electromagnetic surface wave dispersion equation for magnetized dusty e^+e^- plasmas:

$$\sqrt{k_z^2-\frac{\omega^2}{c^2}\left(1-\frac{\omega_{p+}^2+\omega_{p-}^2}{\omega^2-\omega^2}\right)}+\sqrt{k_z^2-\frac{\omega^2}{c^2}\left(1-\frac{\omega_{p+}^2+\omega_{p-}^2}{\omega^2-\omega^2}\right)}=0. \tag{23}$$

Obviously, Eq. (23) can be derived from the well-known dispersion relation for the magnetized cold ion-electron plasmas [Eq. (68) in Ref. [22]] by making the correspondence $\omega_{pi}^2\to\omega_{p+}^2$, $\omega_{pe}^2\to\omega_{p-}^2$, and $\Omega_i=\Omega_e\to\Omega$. Equations (18) and (21) give the jump in E_x

$$[E_x]\equiv E_x(0^+)-E_x(0^-)=\frac{ack_z}{\omega}\left\{\frac{\omega^2-\Omega^2}{\omega^2-\Omega^2-\omega_{p+}^2-\omega_{p-}^2}-1\right\}. \tag{24}$$

Equation (24) shows that the normal field component E_x is discontinuous across the interface and this discontinuity is due to the cold electron and positron components which form a surface charge in a thin layer near the interface. Equations (19) and (22) show the fact that the field component B_y is continuous across the interface in cold magnetized e^+e^- plasmas. The linear electromagnetic surface wave dispersion Eq. (23) for magnetized dusty e^+e^- plasmas can be solved analytically with the aid of some numerical work and using dimensionless parameters

$$\overline{K}\equiv c^2k_z^2/\omega_{p-}^2,\quad \overline{\omega}\equiv\omega^2/\omega_{p+}^2,\quad \overline{\Omega}\equiv\Omega^2/\omega_{p+}^2,\quad \delta\equiv n_{o+}/n_{o-}.$$

Equation (23) can be rewritten using this parameter

$$\sqrt{\overline{K}-\overline{\omega}\left(1-\frac{1+\delta}{\overline{\omega}-\overline{\Omega}}\right)}+\sqrt{\overline{K}-\overline{\omega}\left(1-\frac{1+\delta}{\overline{\omega}-\overline{\Omega}}\right)}=0. \tag{25}$$

where δ is the charge imbalance parameter in the plasma with the remainder of the charge residing on the dust particle. Squaring Eq. (25) yields the following solutions by quadruture

$$\overline{\omega}=\frac{1}{2}\{2\overline{K}+\overline{\Omega}+1+\delta\pm\sqrt{(2\overline{K}+\overline{\Omega}+1+\delta)^2-4\overline{K}(2\overline{\Omega}+1+\delta)}\}. \tag{26}$$

All the solutions of Eq. (25) are included in Eq. (26), but some of the solutions in Eq. (26) are spurious as they are

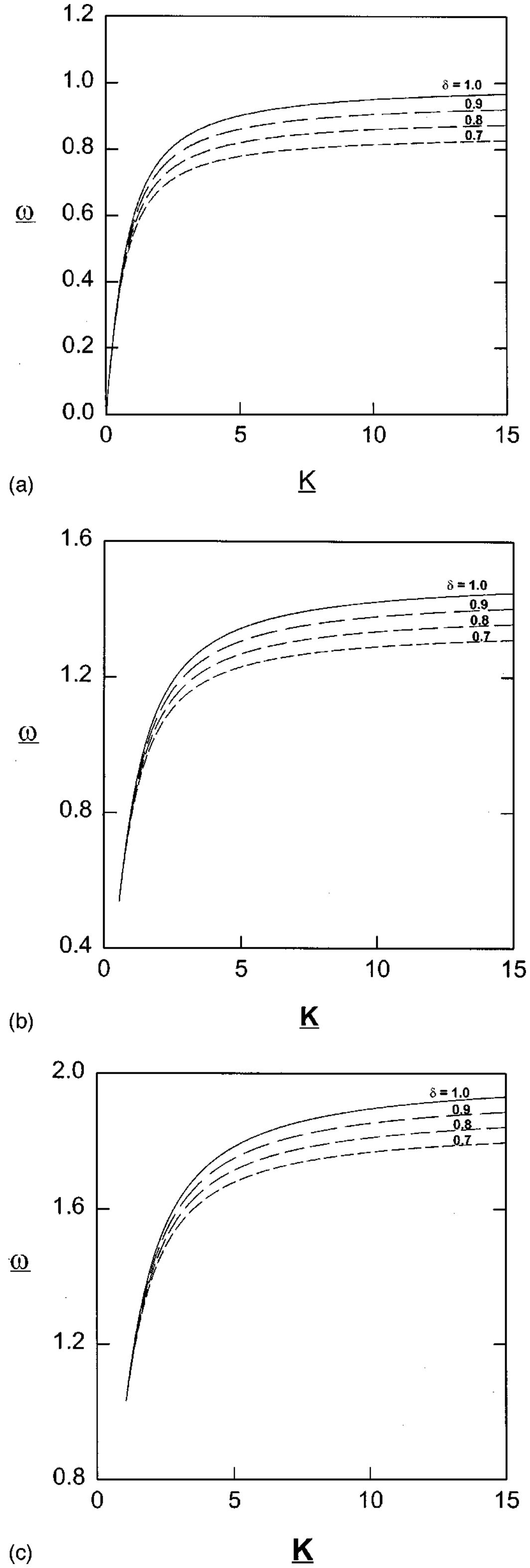

FIG. 1. Electromagnetic surface wave dispersion curves in magnetized cold dusty e^+e^- plasmas for several values of charge imbalance parameter δ. (a) $\bar{\Omega}=0$, (b) $\bar{\Omega}=0.5$, and (c) $\bar{\Omega}=1.0$.

generated in the process of squaring. we take the genuine solutions by checking against the original solution, Eq. (25). The (+) sign in Eq. (26) should be rejected, giving only following equation:

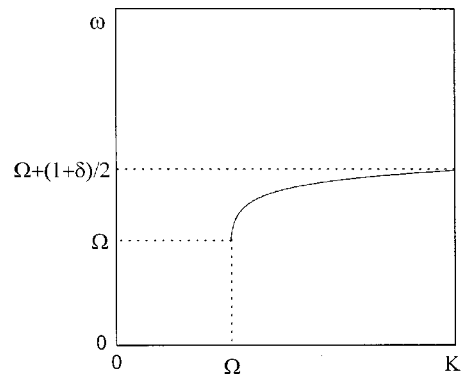

FIG. 2. General feature of the electromagnetic surface wave dispersion curve in magnetized dusty e^+e^- plasmas.

$$\bar{\omega}=\frac{1}{2}\{2\bar{K}+\bar{\Omega}+1+\delta - \sqrt{(2\bar{K}+\bar{\Omega}+1+\delta)^2-4\bar{K}(2\bar{\Omega}+1+\delta)}\}. \quad (27)$$

Equation (27) is the analytical dispersion relation equation of nonrelativistic electromagnetic surface wave in a magnetized dusty e^+e^- plasmas. In Figs. 1(a)–1(c), we plot the electromagnetic surface wave dispersion curves in magnetized e^+e^- plasma for various different values of charge imbalance parameter δ and normalized cyclotron frequency $\bar{\Omega}$, as numerically obtained. when the parameter δ is increased toward 1, this dispersion curves (solid lines) which is indicated in Figs. 1(a)–1(c) are identical with our earlier result of dust-free case [25]. The normalized frequency is decreased as the effect of dust particle is increased. Also, these dispersion modes show that both $\bar{K}$ and $\bar{\omega}$ have the restricted range in order to exist the electromagnetic surface waves in magnetized dusty e^+e^- plasmas. In order to investigate this dispersion curves precisely, the general behaviors of the function $\bar{\omega}$ are indicated in Fig. 2. we found following interesting results in the processing of numerical work. The following results tell us the condition for the waves to be electromagnetic surface wave in the presence of magnetic field by inspecting Eqs. (25) and (27)

$$\bar{\Omega}<\bar{K}<\infty, \quad (28)$$

$$\bar{\Omega}<\bar{\omega}<\bar{\Omega}+(1+\delta)/2. \quad (29)$$

Inequalities (28) and (29) indicate that the allowed ranges of wave-number vector $\bar{K}$ and frequency $\bar{\omega}$ are restricted by external magnetic field effect. These inequalities also show that the normalization frequency $\bar{\omega}$ approaches the value $\bar{\Omega}+(1+\delta)/2$ as normalization wave vector $\bar{K}$ goes to infinity, and the electromagnetic surface wave does not appear for the region $\bar{K}<\bar{\Omega}$.

IV. SUMMARY

In this paper, we have shown that the problem of TM-mode electromagnetic surface wave modes for an dusty e^+e^- plasma can be investigated by specular reflection procedure. In the case of magnetized cold plasmas, we have shown that the field component E_z and B_y are continuous across the interface, but the normal field component E_x is discontinuous across the interface. This discontinuity takes place because of the cold electron and positron components which form a surface charge in a thin bounded layer. Under the consideration of an dusty particle effect, the normalized frequency is decreased compared with that of dust-free case $(\delta=1)$. The most remarkable results are that the electromagnetic surface wave dispersion relation for the cold magnetized e^+e^- plasma shows that the possible surface wave modes appear only when the frequency $(\overline{\omega})$ and the wave vector $(\overline{K})$ satisfy the condition $\overline{\Omega}<\overline{\omega}<\overline{\Omega}+(1+\delta)/2$ and $\overline{\Omega}<\overline{K}<\infty$. Consequently, Eqs. (28) and (29) tell us the conditions for the waves to be TM-mode electromagnetic surface waves in cold magnetized e^+e^- plasmas.

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